Resonances near an energy-level crossing

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We consider a 2×2 matrix system, the diagonal part of which consists of semiclassical Schrödinger operators, and the off-diagonal of a lower order differential interaction. We assume that the two potentials cross transversally with value 0, and that, at this energy level, one of the two potentials admits a well, while the other one is non-trapping.

If the interaction is absent, this matrix operator has, in a neighborhood of 0, eigenvalues created by the potential well, embedded in the continuous spectrum. If the interaction is present, one expects that these eigenvalues are replaced by closely located resonances in the lower complex plane. The imaginary part of resonances is related with the connection problem of solutions at the crossing point.

We compute the quantization condition and study the asymptotic distribution of these resonances $E = E(h)$. In particular, we show that the imaginary parts of the resonances behave exactly like $h^{5/3}$, except possibly for particular values of the limit $h^{-2/3}E(h)$ corresponding to positive zeros of some Airy-type function, and for which $\operatorname{Im} E(h) = o(h^{5/3})$.

This is a joint work with André Martinez (Bologna University) and Takuya Watanabe (Ritsumeikan University).