On mathematical background of practicalization of computerized tomography at its initial stage

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In this talk, we introduce the idea by G.N. Hounsfield for the practicalization of the computerized tomography (CT). Godfrey Newbold Hounsfield (1919-2004) was an English electrical engineer. He developed an idea to practicalize CT, for which he shared the 1979 Nobel Prize for Physiology or Medicine with Allan McLeod Cormack. It being a pity that G.N. Hounsfield left no paper on his idea to practicalize CT, in this talk, we shall introduce what is believed to be his idea.

G.N. Hounsfield considered a simple discretization of the mathematical model for CT and reduced the problem of CT to a problem to solve a system of linear equations having no solution. As a simple example of such systems, consider the system consisting of the following six linear equations.

$$x + y = 3,\ x + y = 1,\ x = 2,\ x = 0,\ y = 2 \text{ and } y = 0. \tag{1}$$

Though this system trivially has no solution, we claim that the solution to (1) is $x = y = 1$, which is by the idea of the least square solution. In the treatment of CT, however, the system would consist of more than 10^7 variables and more than 10^9 equations, which cannot be solved numerically with application of the sweeping-out method for the matrices by computers.

The essence of the idea by G.N. Hounsfield is how to obtain such a system of linear equations having no solution and how to solve it, to introduce which is one of our main purposes in this talk. Although his idea works well in practice, almost all procedures to reconstruct the section of the human body are not theoretically justified. It is the other main purpose in this talk to discuss how to prove these procedures to be right.

References

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