Computational explorations in modern number theory: the Green–Tao theorem and the abc conjecture

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We present a hands-on example of computational thinking at the intersection of mathematics and programming. Using the **Julia programming language and its Pluto.jl notebook** environment, we visualize exploratory computations inspired by three central themes in modern number theory: the Green-Tao theorem, the abc conjecture, and the Collatz conjecture. By combining built-in primality tests with compact code written in **Julia**, **Python**, **MATLAB**, and **Mathematica**, we generate long arithmetic sequences of primes and enumerate abc-triplets with unusually small radical values. Our educational objective is to allow students to experience the scale and subtlety of modern number-theoretic phenomena through interactive and reproducible computation.

GitHub

https://github.com/fiomfd/ATCM2025

Interactive notebooks of Julia & Pluto, Python & Jupyter Notebook and MATLAB livescript are available:

- **Number Theory** (this talk): the prime number theorem, the Green-Tao theorem, the abc conjecture, the Collatz conjecture
- **Calculus I**: differentiability and tangent lines, taylor expansion, Riemann sum
- **Calculus II**: tangent planes, 2D and 3D polar coordinates, Newton's method vs gradient descent, Riemann sum
- **Data Analysis**: public data (HK Observatory) and visualization, central limit theorem
- **Fourier Analysis**: basic theory of Fourier series and its applications (Weyls's equidistribution theorem, continuous but nowhere differentiable functions), Shannon's sampling theorem and aliasing
- **Linear Algebra**: grayscale and RGB images, grayscale and RGB movies, SVD and low rank approximation, Haar wavelet decomposition

Jupyter Notebook at Google Colab

https://colab.research.google.com/github/fiomfd/ATCM2025/

You can run all the above Jupyter Notebooks of Python on the web.
Your Google ID is required. No installation is required.

Try it now on your laptop, tablet or mobile phone!